transcribing the language; groupings of consonants rarely occur, in T. frequently. *One simple consonant* generally heads the word and syllable; names for colors begin with *a-* or *os-*. The parts of the human frame mostly terminate in *-to, -co, -no, -son;* others begin with *oko-, okun-,* which would not be altogether disparate from the T. akon *man,* provided it has the same signification. It is true that the Caddo: nishe *moon,* aχkóto *cold, winter* bear some external analogy with the T. na-ashod *moon* and atsoχ *cool,* but there is often a wide difference between resemblance and real affinity. I have given elsewhere a collection of T. words resembling to terms of the surrounding idioms, which might be augmented indefinitely. Only one of the compared languages has yielded a few terms resting probably on real identity, and that is *Aztec.* We find T. (ya-) χa *to eat,* ka (-la) *mouth* and Aztec (tla-) ka, ka (-matl); T. hauei, auvei *great, large,* Azt. vei, huei; T. aχ *water* and Azt. atl. If these coincidences, which Tonkawa has in common with many Sonora languages, are increased by others, we may look out for proofs of old connections between the two ethnical bodies; connections through commerce, expeditions, emigrations or immigrations, not as yet through ethnological affinity. Up to this day a kinship of the Tonkawas with any other American nation or tribe has not been shown, and neither Aztecs, nor Shoshones or Caddoes can claim it on linguistic grounds. A faint resemblance could be traced in two Caddo terms only and phonology as well as grammar disagrees in most particulars from that of the Tonkawas.

NOTE.—Remember well, that the χ used here has *nothing to do* with the English *x,* but represents the harsh, guttural aspirate *kh* unknown to the English language.

On the Atmospheres of the Sun and Planets.

BY DAVID TROWBRIDGE, A. M.

Read before the American Philosophical Society, November 3, 1876.

There are two cases to be considered; in the first place we may suppose the body surrounded with an atmosphere, to be so hot as to influence, to a considerable extent, the density of the circumambient fluid; and in the second place we may regard the planet as cold like the earth, and as we suppose, Mercury, Venus, and Mars. I shall suppose the atmospheres composed of gases which are subject to the same laws as terrestrial gases.

1. Let us suppose the solar or planetary body to be a sphere of radius r. Also let z be the hight of any stratum of the atmosphere above the surface of the planet; ρ the density of that stratum; Z the force of gravity at the hight z; g the force of gravity at the surface of the planet; $\triangle$ the density of the atmosphere at the surface of the planet; p the pressure at the hight z; P at the surface; t_0 the temperature above 32° Fah. at the surface, and t at the hight z; λ the coëfficient of expansion; or the fraction expressing

the increase in volume for an increase in temperature of 1° Fah., so that if V_0 be any volume at the temperature of 32°, it will become $V_0 (1 + \lambda t_0) = V_1$ at the temperature t_0 above 32°; and at the temperature t above 32° the volume will be

$$V_0 (1 + \lambda t) = V_1 \left(\frac{1 + \lambda t}{1 + \lambda t_0}\right) = V \ldots\ldots (1).$$

Since the pressure is equal to the weight, we have

$$dp = \rho Z \left(\frac{1 + \lambda t}{1 + \lambda t_0}\right) dV_1 \ldots\ldots (2).$$

This equation applies for the hight z. We have

$$V_1 = \tfrac{4}{3} \pi [(r + z)^3 - r^3], \text{ and } dV_1 = 4 \pi (r + z)^2 dz \ldots\ldots (3).$$

$$Z = \frac{gr^2}{(r + z)^2} \quad (4), \qquad t = \frac{r^2 t_0}{(r + z)^2} \ldots\ldots (5).$$

Mariotte's Law gives

$$p = \frac{P}{\triangle}\rho, \qquad dp = \frac{P}{\triangle} d\rho \ldots\ldots (6).$$

If we substitute these values in (2), and make the second member negative, since ρ decreases as z increases, we shall have

$$\frac{1}{\rho} d\rho = - 4 \pi gr^2 \frac{\triangle}{P(1+\lambda t_0)} \left[1 + \frac{r^2 \lambda t_0}{(r + z)^2}\right] dz \ldots\ldots (7).$$

The integral of this gives

$$\text{Log. } \rho + C = - 4 \pi gr^2 \frac{\triangle}{P(1+\lambda t_0)} \left[z - \frac{r^2 \lambda t_0}{r + z}\right]$$

When $z = 0$, $\rho = \triangle$, and $C + \log. \triangle = 4 \pi gr^2. \dfrac{r \triangle \lambda t_0}{P(1+\lambda t_0)}$, and

$$\text{Log. } \frac{\rho}{\triangle} = - 4 \pi gr^2. \frac{\triangle}{P (1+\lambda t_0)} \left[z + \lambda r t_0 - \frac{r^2 \lambda t_0}{v+r}\right], \text{ or}$$

$$\text{Log. } \frac{\rho}{\triangle} = - 4 \pi gr^2. \frac{\triangle z}{P (1+\lambda t_0)} \left[1 + \frac{r \lambda t_0}{r+z}\right] \ldots\ldots (8).$$

Now make

$$\frac{4 \pi gr^2 \triangle}{P} = \frac{1}{R} \ldots\ldots (9);$$

then we have

$$R \log. \frac{P}{\triangle} = - \frac{z}{1 + \lambda t_0} \left[1 + \frac{r \lambda t_0}{r + z}\right] \ldots\ldots (10).$$

Let us take the surface of the earth as the unit of surface, the radius of the earth as a linear unit, and the force of gravity at the earth's surface as the unit of gravity.

2. Let D' be the density of mercury at the temperature of 32°, and D'' its density at t_0 above 32°; then, if λ' be the coëfficient of expansion for a volume of mercury,

$$D' = (1 + \lambda' t_0) D'' \ldots\ldots (11).$$

Let h' be the hight of a column of mercury on the unit of surface, and at the temperature of 32°, that will just balance a column of atmosphere

on the same surface, h'' the hight of a column of mercury on the unit of surface on a planet, at the temperature t_0 above 32° ; then the mass of the first column will be $4\pi h'D'$ very nearly since h' is about 30 inches, and the weight will also be $4\pi h'D'$, and this will be equal to P_0 the pressure on the unit of surface on the earth. Hence

$$P_0 = 4\pi h'D' \qquad (12).$$

The mass of the second column will be

$$4\pi h''D'' = \frac{4\pi h''D'}{1 + \lambda' t_0} \qquad (13);$$

and the weight will be $\frac{4\pi h''gD'}{1+\lambda' t_0}$, which is equal to the pressure P on the unit of surface. Hence

$$P = \frac{4\pi h''gD'}{1 + \lambda' t_0} \qquad (14).$$

So long as the temperature is constant the density is proportional to the pressure ; or $P_0 : P :: \triangle_0 : \frac{P\triangle_0}{P_0}$; and under different circumstances of temperature the density is inversely as the volume for the same mass, so that for the temperature t_0 above 32°, the density just found will become $\frac{P\triangle_0}{(1 + \lambda t_0)P_0}$, and this must represent the actual density $\triangle$ at the surface of the planet. Hence

$$\triangle = \frac{\triangle_0}{(1 + \lambda t_0)} \cdot \frac{P}{P_0} = \frac{4\pi h''gD'}{4\pi h'D'(1+\lambda' t_0)} \cdot \frac{\triangle_0}{(1+\lambda t_0)} = \frac{\triangle_0 g h''}{h'(1+\lambda' t_0)\,(1+\lambda t_0)} \qquad (15).$$

3. Let *m* be the mass of any one of the planets, that of the Earth being 1, *A* the mass of the Earth's atmosphere, and KAm that of the planet's, then

$$A = 4\pi h'D' \qquad (16) ;$$

$$\text{and } KAm = \frac{4\pi h''D'r^2}{1 + \lambda' t_0} \qquad (17) ;$$

and hence

$$\frac{KAm}{r^2} = KAg = 4\pi Kgh'D' = \frac{4\pi h''D'}{1 + \lambda' t_0}, \text{ or}$$

$$h'' = Kh'g\,(1 + \lambda' t_0) \qquad (18).$$

This value in (15) gives

$$\frac{\triangle}{\triangle_0} = \frac{Kg^2}{1 + \lambda t_0} = \frac{P}{P_0\,(1 + \lambda t_0)} \qquad (19).$$

Equation (10) is independent of the extent of surface on which the atmosphere presses, since it gives only the law of the variation of density of the atmosphere in ascending through it. R is therefore a linear measure, and on the Earth it has been found from observations to be 26,126.5 English feet, or 4.948 English miles. To compare this with a similar quantity for any one of the planets, we must make $r = 1$ in Eq. (9), and we shall make

$$4\pi R_0 = \frac{P_0}{\triangle_0} \qquad (20), \qquad 4\pi gR' = \frac{P}{\triangle} \qquad (21),$$

in which $R_0 = 4.948$ miles. These equations give

$$\frac{gR'}{R_0} = \frac{P}{P_0} \cdot \frac{\triangle_0}{\triangle} = 1 + \lambda t_0 \ldots\ldots (22).$$

In Eq. (10) we must therefore substitute for R the value $R' = \frac{(1+\lambda t_0)R_0}{g}$, which gives

$$R_0 (1 + \lambda t_0)^2 \log. \frac{\rho}{\triangle} = - gz \left[1 + \frac{r\lambda t_0}{r+z}\right] \ldots\ldots (23).$$

Equation (23) is independent of K, which depends on the mass of the planets' atmosphere, as it should be since we have neglected the attractive influence of the atmosphere. This seems allowable since the mass of the Earth's atmosphere is but a small fraction of the mass of the Earth.

4. Now let $t_0 = 0$ in (23) and the resulting equation will apply to an atmosphere of 32° Fah.

$$R_0 \log. \frac{\rho}{\triangle} = - gz \ldots\ldots (24).$$

To make some application of these formulas we shall take $g = 1$ and $\triangle = 2\rho$, and since N log. $2 = 0.6931472$ we find $z_1 = 3.4297$ miles ; and for $\triangle = 4\rho$ we have $z_2 = 6.8594$ miles, and so on, $\triangle = 2^n\rho$ giving $z_n = nz_1$.

Now let us suppose $g = 1$ and $\lambda t_0 = 1$, then, $\triangle = 2\rho$, $4z_1 = z + \frac{rz}{r+z} = \frac{z^2 + 2rz}{r + z}$, and $z^1_1 = -(r - 2z_1) \pm \sqrt{r^2 + 4z_1^2}$.

We must reject the negative value, and for the other we have

$$z'_1 = 2z_1 + \frac{2z_1^2}{r} \text{ very nearly, } = 6.8649 \text{ miles.}$$

In a similar manner we find $z'_2 = 13.7425$, or $z'_2 = 2z'_1 + 0.0127$ miles. We should also find $z'_3 = 3z'_1$, very nearly, for $\triangle = 3\rho$; and so on.

We thus see that if the Earth were 490° Fah. warmer than it now is $\left(\text{since it is found that } \lambda = \frac{1}{490}\right)$; or in other words, if its own heat was such as to heat the air in contact with it to 490° more than 32° (supposing it to be 32° now), and to vary according to the law which we have supposed, the density of the atmosphere would be one-half what it is at the surface at the hight of about 7 miles, instead of $3\frac{1}{2}$ as at present ; and one-fourth the density would be reached at about 14 miles, and so on. The hight of the atmosphere under such conditions would be more than double what it now is. If we suppose the density at the surface of the Earth to be about a million times as great as at the surface of the atmosphere, or $\triangle = 2^{20}\rho$, we shall find $z = 139$, nearly, under the conditions of high temperature ; while in the other case it will be but 69 miles.

For the temperature which we have supposed the Earth to have, it would scarcely give out any light. To suppose the body self-luminous, it will be necessary to make t_0 about twice what we have supposed. We shall now make an application of our formulæ to the planet Jupiter, and since that

planet seems to be brighter, or to give out more light than what it reflects from the Sun, we shall suppose λt_0 equal to 2. The force of gravity on Jupiter is about 2.42 times terrestrial gravity; and if we make some allowance for an extensive Jovian atmosphere we may perhaps call $g = 2.5$.

If these values be put in Eq. (23), and we make $\triangle = 2\rho$, we shall have $3.6z_1 = \frac{z^2 + 3rz}{r + z}$, and $z = 1.2z_1$ nearly, $= 4.1156$ miles.

If we make $t_0 = 0$ we shall find $z = 1.3718$ miles, or about one-third of the other value. In a similar manner we shall find for the relation $\triangle = 4\rho$, $z = 8.2313$ miles, and so on.

If in Equation (19) we make $K = 1$, and $\lambda t_0 = 2$, we shall have $\triangle = 2\triangle_0$ ..(25) very nearly. Thus at the hight of about 4 miles the density of the Jovian atmosphere will, upon these suppositions, be the same as the density of the Earth's atmosphere at the surface of the Earth. If we suppose, as Mr. R. A. Proctor has done, that the density of Jupiter's atmosphere where the cloud-layers exist, is one-fourth of the density of the Earth's atmosphere at the Earth's surface, we shall have z for that case about 12 miles instead of 100 as he supposes. According to this investigation, then, Jupiter's atmosphere at his surface is either very dense, or its extent is not so great as Mr. Proctor imagines; or his temperature at the surface is greater than 1000° Fah.

We may perhaps reasonably assume $\triangle = 2^{50}\rho$ on Jupiter, and this value will give the hight of his atmosphere about 206 miles; and if we suppose $\triangle = 2^{20}\rho$ we shall find z equal to about 82 miles.

If we call $K = 1$, $t_0 = 980°$, $\lambda' = \frac{1}{9800}$, $g = 2\frac{1}{2}$, we shall find, by Eq. (18), the hight of the barometer on Jupiter $82\frac{1}{2}$ inches. By Equation (19), we find upon the same suppositions that the pressure to the square inch, of Jupiter's atmosphere on his surface, is equal to $93\frac{3}{4}$ pounds.

If we take for Saturn $g = 1.1$, $\lambda t_0 = 2$, $\triangle = 2\rho$, we shall find $z = 9.3537$ miles; and for $\triangle = 2^{20}\rho$, $z = 187$ miles; and for $\triangle = 2^{50}\rho$, $z = 468$ miles. If we take $K = 1$, Eq. (19) gives $\triangle = \frac{2}{5}\triangle_0$, and the pressure to the square inch equals 18 pounds. The hight of the barometer by Eq. (18) equals 36 inches.

To make some application to the Sun, I shall assume $\lambda t_0 = 49$, which will give a temperature of the Sun's surface sufficiently high to correspond with our knowledge of the subject. We also have g equal to 27.2; and if $\triangle = 2\rho$ we find $z = 6.304$ miles. If we suppose $\triangle = 2^{1000}\rho$, we shall have $z = 6304$ miles as the corresponding hight of the solar atmosphere. We also find $\triangle = 15\triangle_0$ very nearly, if $K = 1$; and the pressure to the square inch is about 11000 pounds. The hight of the barometer would be 234.6 feet.

In these applications we have assumed $K = 1$ which supposes the mass of the atmosphere proportional to the mass of the planet which it surrounds. In the absence of all real knowledge on the subject we may as well make that assumption as any. The probability is that this supposition is an ap-

proximation to the truth. Equation (23) is independent of any supposition in relation to this matter.

5. In the following table I have arranged the various elements of the planetary atmospheres, which have been the subject of the foregoing investigation, on the supposition that $K = 1$, and $t_0 = 0$.

Planet.	Value of z for $\triangle = 2\rho$, in Miles.	Density at Surface.	Pressure at Surface on 1 sq. in. in lbs.	Hight of Barometer in inches.	Hight of the Atmosphere in Miles.
Mercury.....	7.40	0.214	3.21	13.9	719
Venus.......	3.92	0.765	11.47	26.2	3[illegible]2
Earth........	3.43	1.000	15.00	30.0	343
Mars........	11 31	0.092	1.38	9.1	1095
Jupiter......	1.42	5.856	87.84	72.6	146
Saturn......	3.12	1.210	18.11	33.0	313
Uranus......	5.20	0.530	7.95	21.8	515
Neptune.....	4.36	0.615	9.22	23.5	436

The last column of this table, or that which gives the hight of the atmosphere, is based upon the supposition that the exterior limits of the planetary atmosphere have the same density for all, and that the Earth's atmosphere has a hight of 343 miles. It is not a little curious at first sight that Jupiter's atmosphere should be the least extensive in hight, while Mars's is the most extensive; but this is due to the great attractive influence of the one, and the smallness of the same element in the other. Though this table is based on an uncertain hypothesis, yet it seems to me that it is considerably instructive. Besides, the preceding theory will serve to test some of the hypotheses which are offered to account for the surface appearances of Jupiter.

The peculiar figure which Sir William Herschel found Saturn to present, which Bessel and others could not discover, and which Airy could not explain upon the theory of gravitation, is accounted for without difficulty by the currents in his atmosphere, as will be seen by consulting Wm. Ferrel's *Treatise on the Motion of Fluids relative to the Earth's Surface*, Art. 18, Fig. 1, Vol. I, *Math. Monthly;* and if Jupiter's atmosphere were sufficiently extensive he ought to present a similar figure.

Waterburgh, N. Y., October 19, 1876.

NOTE.

Since the above was written I have seen a report of Prof. S. P. Langley's experiments to determine the temperature of the solar surface. Physicists have heretofore differed considerably in their estimates of the Sun's temperature, Secchi putting it as high as 18 millions of degrees Fah., St. Claire Deville and others, between 3,000° and 20,000°. Prof. Langley's result favors the higher number. For convenience of calculation let us assume the solar temperature equal to 9,800,032° Fah., so that $\lambda t_0 = 20{,}000$.

Now let $K = 1$, then we find, by Equation (19), $\triangle = 0.0369 \triangle_0$; or the

density of the solar atmosphere at the surface of the Sun, is but little more than one-half the density of hydrogen at the surface of the Earth, and at a temperature of 60° Fah. The pressure on a square inch of surface, is about 11,000 pounds Avoir. In the case of the Earth the density of the atmosphere at the surface of the Earth, is to the density at the surface of the atmosphere, at least as great 2^{100}: 1. If we assume the same ratio to hold for the Sun's atmosphere, Eq. (23) will give us for the hight of the solar atmosphere, 426,000 miles, or about the radius of the Sun. If we make $t_0 = 0$, we shall find the hight of the atmosphere, with the same ratio of densities, about $12\frac{1}{2}$ miles; the density at the Sun's surface about 740 times that of the Earth's atmosphere at the surface of the Earth, or nearly equal to the mean density of the Sun. It will be noticed that in the case of high temperature, if we assume $K = 10$ (a very improbable value, it being too large), we still have $\triangle = 0.369_0 \triangle_0$, which gives a rare atmosphere. The following table will give the density at different hights above the Sun's surface:

Relative Density.	Hight in Miles.	Absolute Density, or $\triangle_0 = 1$.
$\triangle = 1$	0	0.03699
$\frac{1}{2}$	2536	0.01849
$\frac{1}{4}$	5104	0.00924
$\frac{1}{8}$	7702	0.00462
$\frac{1}{16}$	10330	0.00231
$\frac{1}{32}$	13000	0.00116
$\frac{1}{64}$	15683	0.00058
$\frac{1}{128}$	18403	0.00029
$\frac{1}{256}$	21362	0.00014

These numbers will help us to explain the rapid movements which Prof. C. A. Young and others have noticed in the solar atmosphere. If gases are pent up beneath the solar surface, but finally escape with great force, the rare atmosphere of the sun would not retard the motion like a dense one. The less rapid motion may be due to difference of specific gravity. The density of the hydrogen clouds can be calcnlated from the formulæ which have been given in the text. D. T.

November 24, 1876.

[*Minutes continued from page* 292.]

The minutes of the last meeting of the Board of Officers and Members in Council were read, and the recommendations of the Board were acted upon as follows:

A committee of five, to be appointed by the Chairman, after consultation, was, on motion of Mr. Price, ordered, who should accept Mr. Wooten's invitation, and examine his anthracite slack fires at Reading, or elsewhere along the line of the Reading Railroad.